

**Figure 6:** The fundamental derivative for cases 1 to 3 (MUSCL VFRoe, 500 cells, at the time instants corresponding to the results shown in Figures 3, 4 and 5).

## **CONCLUSIONS**

Some Riemann solver-based methods were compared for two ideal gas flow problems. Due to the difficulty in application of Riemann solvers to real gases, three Riemann solver-free methods were compared for three real gas flow problems taken from the literature. The methods were compared not only for a given mesh, but the performance for a given accuracy with different meshes was also analyzed. This methodology allows the correct evaluation of the advantages of the MUSCL strategy for achieving second-order accuracy.

Several issues were analyzed for the first time in the literature: the comparison of the recent proposed hybrid Lax-Friedrich-Lax-Wendroff (De Vuyst, 2004) method with the most popular hyperbolic solvers, the MUSCL second-order extension of the VFRoe method and the application of these methods to real gases flows. Besides, an accurate procedure for imposing the boundary condition when there are source terms was clearly presented and compared to common procedures in a pipeline shutting-in problem.

The Riemann solvers of Roe-Pike, Harten and the MUSCL-Hancock MINMOD and SUPERBEE second-order extensions of the Roe-Pike method were compared for accuracy and CPU time cost using ideal gas flows in shock tube and pipeline shutting-in test problems. The CPU cost was also evaluated in terms of the number of discretization cells, time steps and variables. The results show that, although the MUSCL-Hancock second-order extension requires more FLOP's for a given mesh

and CFL number, it is faster than first-order methods because, for a given accuracy, it requires fewer cells and consequently allows larger time steps than the first-order methods. It was shown that the Roe-Pike solver with MUSCL-Hancock extensions are more accurate and faster than the original Roe-Pike and Harten methods. The SUPERBEE MUSCL scheme presented numerical oscillations near discontinuities common to second-order methods whereas the MINMOD MUSCL scheme did not.

The hybrid Lax-Friedrich-Lax-Wendroff, VFRoe and AUSM+, the latter two in their MUSCL-Hancock second-order extensions, were applied to real gases using the Van der Waals EOS. The accuracy and CPU cost of these methods were compared for three test cases. In one of the tests the MUSCL VFRoe and the hybrid method presented large errors due to entropy violation. The MUSCL and VFRoe schemes showed small oscillations near discontinuities for one of the test cases, also producing strange oscillations in two adjacent constant value regions. This was shown to be caused by the MUSCL second-order extension. The MUSCL AUSM+ scheme was always much faster than the others. The MUSCL VFRoe method was the slowest scheme for large meshes, but it was faster than the hybrid method for small meshes with comparable accuracies.

Therefore, the AUSM+ scheme is generally recommended for compressible gas-phase flow problems, although caution must be taken in the use of its MUSCL extension due to strange nonphysical oscillations for some real gas flows.